MODEL REFERENCE ADAPTIVE SYSTEMS

Amin Suyitno
Fakultas Sains dan Teknologi, Universitas Buddhi Dharma
Jalan Imam Bonjol No. 41, Tangerang, Indonesia
Email: amin.suyitno@buddhidharma.ac.id

Abstract
The most widely used electrical machines in industry for a variety of modern high-performance drive applications are induction motors (IMs), particularly those using squirrel cage rotors. To realize the control of sensorless IM drive and achieve high precision, proper knowledge is required to obtain adequate IM parameter values. [3] Such as obtaining the stator resistance and rotor resistance values of the IM used, the variation of which can cause inaccurate speed estimation over the entire operating range of the sensorless drive. A number of methods for high-performance sensorless IM drive speed estimation by considering the influence of parameter variations have been developed [1],[11]. This paper presents the Model Reference Adaptive Control (MRAC) method, in this control method the plant parameters are not specified. The appropriate model is selected and used to design the controller. Therefore, this method has robustness to plant parameter variations. Therefore, this method has robustness to the variation of parameters of the plant. The simulation results show the robustness of this method to parameter variations in the motor drive section and the load in Figures 6 to 9. The simulation results show that the system output is still stable, but the constraint faced in this method is that the controller response is too slow when compared to robust controller types such as the Zeroing method and the VS-MRAC method [9][10]. The controller cannot make a fast response when the load acts on the system. The effect is the output of the system ω_L has a large swing in the transient and it takes a long time to reach a constant value.

Keyword
MRAC, VS-MRAC, Torsional Motor Drive, Flux Vector Control, Disturbance Observer, Zeroing control.

Introduction
The Model Reference Adaptive System (called MRAS) is one of the main approaches to adaptive control [6]. The basic principle is illustrated in Figure 1. The desired performance is expressed in terms of a reference model, which gives the desired response to a command signal. The system also has an ordinary feedback loop composed of the process and the regulator. The error e is the difference between the outputs of the system and the reference model. The regulator has parameters that are changed based on the error. There are thus two loops in Figure 1: an inner loop, which provides the ordinary control feedback, and an outer loop, which adjusts the parameters in the inner loop. The inner loop is assumed to act on the system faster than the outer loop.

The MRAS configuration in Figure 1 was originally proposed by Whitaker at MIT. It was developed around 1958(Butler, 1992)[2], which introduced two ideas:

1. The performance of system is specified by a model.
2. The parameters of the regulator are adjusted based on the error between the reference model and the system.

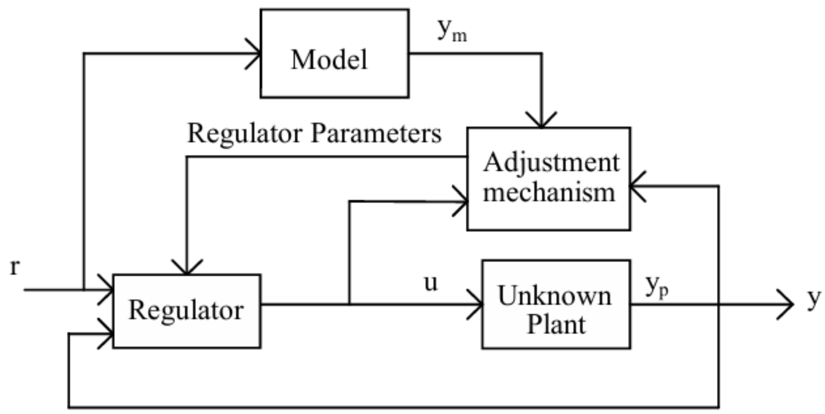

Figure 1. Block diagram of the Model Reference Adaptive System.

These ideas were originally applied to servo motor problems in certain continuous-time systems and recently several methods have been found to extend them.

The Basic Idea of Model Reference Adaptive Controllers

The MRAC used in this research is a kind of adaptive control problem for the caseof an unknown plant with a single-input and a single-output (SISO plant) (Qiang, 2012; Wei et al., 2019)[6][12]; (Singh et al., 2017)[7]. The plant is assumed to be linear and time-invariant. Only the plant input and output are used to generate a control u(t) and the control design is a differentiator-free controller so that the output of the plant evolves asymptotically towards an output of the model. This control algorithm was proposed by Kumpati S. Narendra and Lena S. Valvani [(KUMPATI S. NARENDRA AND LENA S. VALAVANI, n.d.; Kunpati S. Narendra, n.d.)][4][5] and the abbreviated explanations are given in the following chapter. The basic structure of this adaptive control is shown in Figure 2.

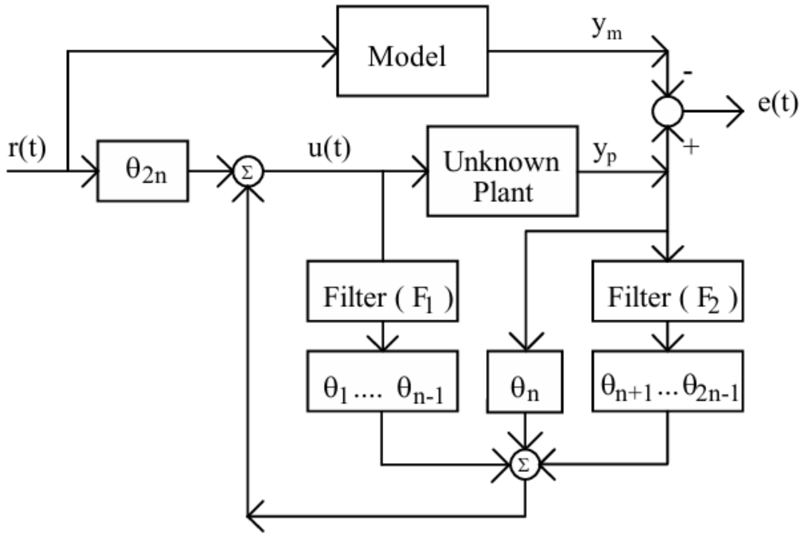

Figure 2. The basic structure of the SISO MRAC.

Overview of the MRAC Design

The plant P, that is to be controlled, is represented by the input-output pair $\{u(t), y_p(t)\}$ and can be modeled by a linear time-variant system described by the state Equations(Suyitno et al., 2024)

$$\dot{x}_p = A_p x_p + b_p u \tag{1}$$

$$y_p = h^T X_p; \quad h^T = [1, 0 \ldots 0]$$

where A_p is an (n x n) matrix, h and b_p are n-vectors. The transfer function $W_p(s)$ of the plant may be represented as

$$W_p(s) = h^T (s\,I - A_p)^{-1} b_p \triangleq k_p \frac{Z_p(s)}{P_p(s)} \tag{2}$$

Let $W_p(s)$ be a strictly proper transfer function with $Z_p(s)$ a monic Hurwitz polynomial of degree $m(\leq n-1)$, and $P_p(s)$ be a monic polynomial of degree n. k_p is a constant gain and the plant input and output are u and y_p.

A model *m* represents the behavior expected from the plant when it is augmented with a suitable controller. The model has a reference input *r*(*t*) and an output $y_m(t)$. The transfer function of the model is denoted by $W_m(s)$, or in the state equation form, may be represented as follows:

$$W_m(s) \triangleq k_m \frac{Z_m(s)}{P_m(s)} \quad \text{or} \quad \dot{x}_m = A_m x_m + b_m u$$

$$y_m = h^T x_m \;;\; h^T = [1, 0 \ldots 0] \tag{3}$$

where k_m is a constant gain, and input and output are r and y_m. The deviation of the plant from the desired behavior is measured by the absolute value of the error between the plant output and the model output as

$$|e_1(t)| \triangleq |y_p(t) - y_m(t)| \tag{4}$$

With the exception of the plant and the controller conditions that we have introduced before, the following assumptions are made:

1. The order of the plant should be known. Supposing it has n poles and m zeros, where $m \leq (n-1)$. The sign of k_p is positive.
2. The order of a defined reference model is the same as the plant order and strictly positive real (called SPR).
3. The plant together with the controller including constant parameters would have a transfer function with (number of poles) - (number of zeros)≥ (n-m).

As shown in Figure 2, this controller has two filters connected to the input andoutput of the plant to represent auxiliary signal generators F_1 and F_2. F_1 and F_2 can be described using respective (n-1)th order vector differential equations.

$$\text{Filter } F_1 = \frac{C(s)}{N(s)} \quad \text{or} \quad \dot{v}_1 = \Lambda v_1 + b_v u$$
$$\text{Filter } F_2 = \frac{D(s)}{N(s)} \quad \text{or} \quad \dot{v} = \Lambda v_2 + b_v u \tag{5}$$

And

$$b_v = [0, 0 \ldots 1]$$

where Λ represents an (n-1)×(n-1) stable matrix. Supposing the values of θ are constant, the transfer function of the plant plus the controller in the closed-loop condition will be

$$W(s) = \frac{\theta_{2n} W_p(s)}{1 + F_1 + W_p(s) + F_2(s) + \theta_n}$$

Or

$$W(s) = \frac{\theta_{2n} k_p Z_p(s) N(s)}{N(s) + C(s) P_p(s) + k_p Z_p(s)\big(D(s) + \theta_n N(s)\big)} \tag{6}$$

$W(s)$ as given in the above equation has (3n-2) poles and (2n+m-2) zeros of which (n-1) are common. It is well explained in the Narendra's paper that a constant parameter vector θ^* exists such that $W(s) = W_m(s)$, which implies there are (n-1) additional pole and zero cancellations. From equation (6-6) it is seen that the poles of the auxiliarysignal generators (F_1 and F_2) are also zeros of $W(s)$.

In order to the overall transfer function $W(s)$ (the plant plus the controller) has the same zeros as the model, the elements of Λ are chosen to contain $Z_m(s)$ as a factor, or in one case $Z_m = det(s\,I - \Lambda)$ can be used. That means, the order of $Z_m(s)$ should be the same as the order of each filter respectively. In the usual adaptive control design, the control signal *u(t)* is constructed as follows:

$$u(t) = \theta(t)^T \omega(t) \tag{7}$$

Where $\theta(t)^T = [\theta_1(t)\ \theta_2(t) \dots \theta_{2n}(t)]$ and the vector ω(t) is defined as $\omega(t) = [v_1^T(t)\ \ y(t)\ \ v_2^T(t)\ \ r(t)]$. The adaptive law used a gradient law, that is

$$\dot{\theta} = -\Gamma e_1 \omega(t), \qquad \Gamma = \Gamma^T > 0 \tag{8}$$

It is well-known that under the above assumption there exists a unique constant Vector θ^* which is produced when the closed loop plant exactly matches $W_m(s)$. The parameters *θ(t)* will adapt to the specific constant values θ^*, and the signal error $e_1(t)$ will gradually become zero as $t \to \infty$. When $\theta(t) \approx \theta^*$, then $u = \theta^* e_1$, the closed-loop transfer function of the plant equals $W_m(s)$. There for output of the plant follow the output of the model.

The Error Equation and the Adaptation Law

In order to obtain the error expression, the state space of the overall system can be represented as

$$\begin{bmatrix} \dot{x}_p \\ \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} A_p & 0 & 0 \\ 0 & \Lambda & 0 \\ b_v h^T & 0 & \Lambda \end{bmatrix} \begin{bmatrix} x_p \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} b_p \\ b_v \\ 0 \end{bmatrix} [\theta^T(t) \quad \omega] \tag{9}$$

$$y_p = h^T x_p$$

Supposing the adaptive parameter θ(t), $\theta(t) = \dot{\theta}(t) + \Phi(t)$, where $\dot{\theta}$ is the constant value when the overall system equals the model, and Φ(t) is the adaptation parameter, then, the state equation (9) will become

$$\dot{x} = Ax + b\left[\dot{\theta}_{2n} r + \Phi(t)^T \omega\right] \tag{10}$$

$$y_p = h^T x_p$$

Where,

$$x^T = \left[x_p \quad v_1^T \quad v_2^T\right]$$

$$A = \begin{bmatrix} A_p + \theta_n^* b_p h^T & b_p \theta_u^{*T} & b_p \theta_y^{*T} \\ b_v \theta_n^* h^T & \Lambda + b_v \theta_u^{*T} & b_v \theta_y^{*T} \\ b_v h^T & 0 & \Lambda \end{bmatrix}$$

$$b^T = [b_p \quad b_v \quad 0]$$

θ_u^* and θ_y^* denoting $\theta_1 \dots \theta_{n-1}$ and $\theta_{n+1} \dots \theta_{2n-1}$ respectively.
Introducing the expression $b = \bar{b}\,\theta_{2n}^*$ so that equation (10) can be rewritten as follows:

$$\dot{x} = Ax + \bar{b}\,[u - \theta^{*T}\omega] + br \tag{11}$$

$$y_p = h^T x_p\ ; \quad h^T = [1 \quad 0 \quad ... \quad 0]$$

Then the error of the system e, is defined as $e \triangleq x - x_m$ Using the model presented in equation (3), we can obtain the error equation as

$$\dot{e} = Ae + \bar{b}\Phi^T\omega\ ; \qquad e_1 = h^T e \tag{12}$$

And the adaptation law equation (8) can be written as

$$\dot{\Phi} = -\Gamma e_1 \omega \tag{13}$$

Understandably, if the system has been adapted so that $\theta(t) \approx \theta^*$ or $\Phi(t) \equiv 0$, then equation (10) and (11) are the same as in the model.

Stability Analysis of the Systems
Mathematically, analysis of the global stability of the adaptive system has been done by Narendra & Valavani 1978, and L. Hsu & Costa 1989. (KUMPATI S. NARENDRA AND LENA S. VALAVANI, n.d.; Kunpati S. Narendra, n.d.)[4][5] When $W_m(s)$ is SPR, then $P = P^T > 0$ and $Q = Q^T > 0$ exist so that

$$A^T P + PA = -2Q, \quad Pb = h \tag{14}$$

When the control signal u and adaptation law is defined by equations (7) and (8), the global stability of the adaptive system can be proved with Lyapunov function

$$2V(e, \Phi) = e^T P e + \frac{1}{\theta_{2n}^*}\Phi^T \Gamma^{-1} \Phi \tag{15}$$

which has form as in equations (12) and (13). Then, the time derivative is as follows:

$$\dot{V} = -e^T Q e \tag{16}$$

Equation (16) can be valid if the elements have (negative) semi-definite in the (e, Φ)-space. It is not asymptotic, but global stability of the origin is implied. Moreover

$$e_1(t) \to 0 \ as\ t \to \infty.$$

Application of MRAC in the Torsional Drive Systems
The torque drive system used in this study has unstable internal dynamic characteristics. This system has mechanical vibrations on its axis, so that the load speed part is not strictly controlled. It is impossible to use the load speed signal as a feedback loop control, even using conventional control designs such as PI or PID. (Strefezza et al., n.d.; Suyitno

et al., 2024)[8] In this chapter, we propose to use the Model Reference Adaptive Control method. By using the desired perfect model, the feedback loop can be constructed from the load speed signal $\boldsymbol{\omega_L(t)}$.

In the explanation of the SISO MRAC, the latter which has been used, it was found that the difference order in s between the zeros and the poles of the plant was only one, but the plant used in this research has a difference order of 2 (see chapter 2). In this case, from reference [6] the specific algorithm was used to solve this difference order. Changing the second assumption that

$$L(s) = (s + \rho) \tag{17}$$

then $L(s)W_m(s)$ is strictly positive real. The value of elements of an (n-1)×(n-1) stable matrix Λ, that is shown in equation (5), may be defined as $L(s)Z_m(s) = \det(sI - \Lambda)$. Thereby, the filters F_1 and F_2 are easily designed. The construction of the control signal u(t) becomes

$$u(t) = \sum_{i=1}^{2n} \left[\theta_i(t) + \dot{\theta}_i L^{-1}\right]\omega_i(t)$$

and the parameters are updated using the law

$$\dot{\theta} = -\Gamma e_1(t)\xi(t)\ ; \quad \Gamma = \Gamma^T > 0 \tag{19}$$

with $\xi(t) = L^{-1}\omega(t)$, It is shown that the error $e_1(t) \to 0$ as $t \to \infty$. Hence, the complete block diagram of the SISO MRAC system becomes,

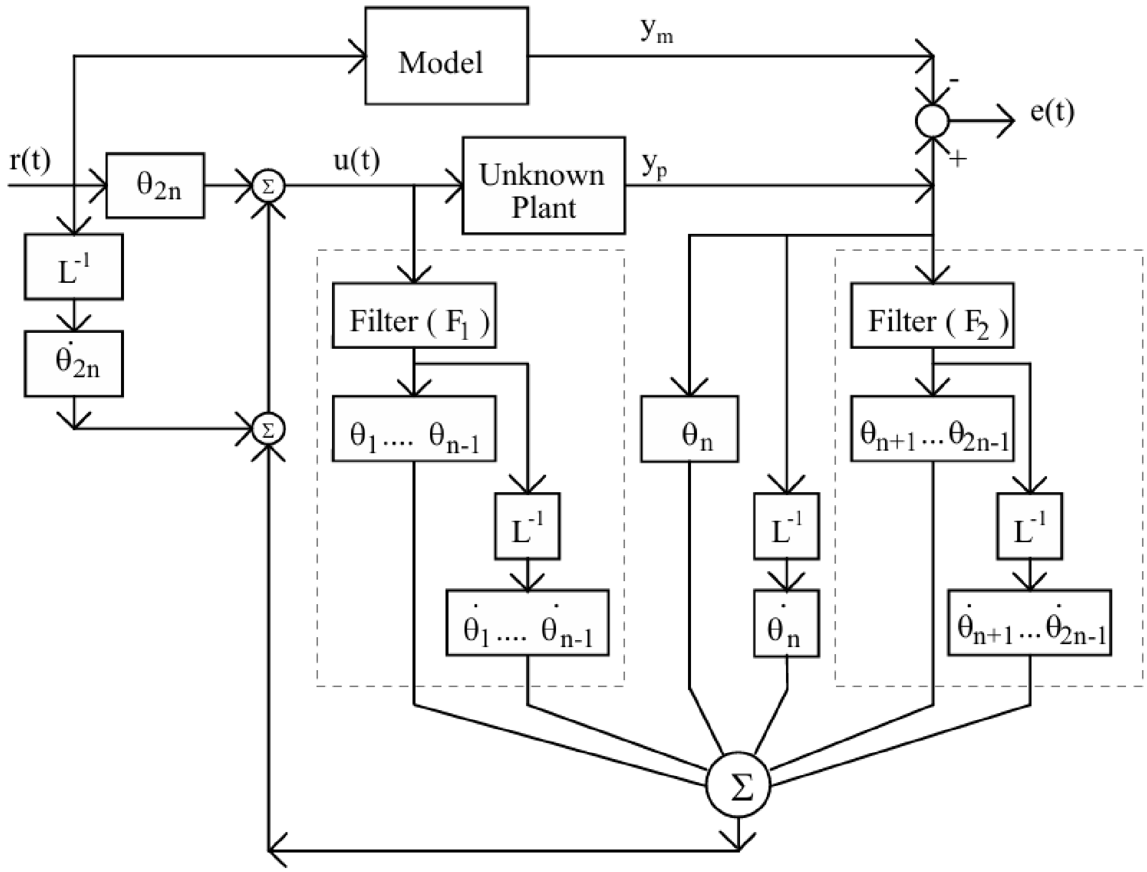

Figure 3. The structure of the complete system using MRAC.

Simulation and Results

In the simulation, the plant used parameters J_m:0.07; J_l:0.17; D_m:0.005; D_l:0.01; D_v:0.5; k_c:500 and k_t:1, which is as before. Thus, using the equation (17) and (18) it is possible to write the exact transfer function or the differential equation of the system. For the necessity of the proper model so that the algorithm can work perfectly, a general third-order system with one-zero is proposed.

$$W_m(s) = \frac{k_m(s+\omega_n^2 p)}{(s^2+2\zeta\omega_n s+\omega_n^2)(s+p)} \tag{20}$$

The filters proposed for use are

$$F_1 = F_2 = \frac{1}{s^2+T_1 s+T_2} \tag{21}$$

and the additional filter L^{-1} is

$$\mathrm{L}^{-1} = \frac{1}{s+T_3}$$

Figure 4 shows the result of the simulation of the system without a load, when the command signal, r = 10 + 10sin(t) with model parameters k_m=1, z=1, w_n=5, p=5, and T_1=2, T_2=1, T_3=0.67 for the filters, where the gain of adaptation law Γ= 0.025 and the simulation used a time sampling of 1 μsec.

In the loaded condition, a 100% load is applied to the system, the simulation result is as shown in Figure 5.a and Figure 5.b shows the error e_1. Figure 5.a shows that output w_L has transients when the loads are changing and the adaptation time needs about 2.5 seconds.

In Figure 7 and 8, the output response of the system, the output of the model (W_{LM}) and the output of the plant (W_L) when the parameters J_M and J_L change to 0.35 and 0.85 respectively are shown. Even though the parameters are increased by a factor of 5, the overall system still stable.

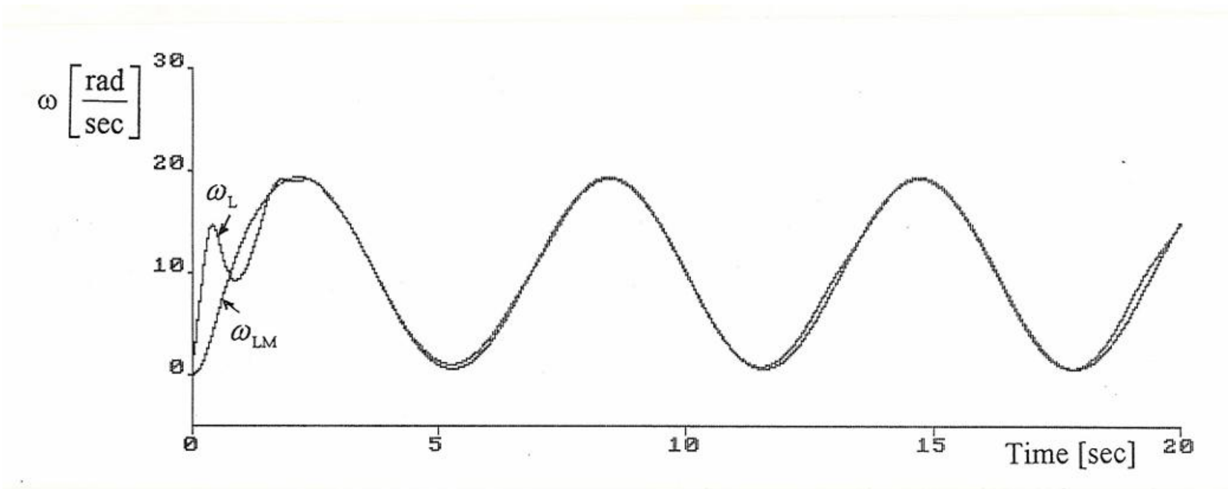

Figure 4. Simulation response without a load.

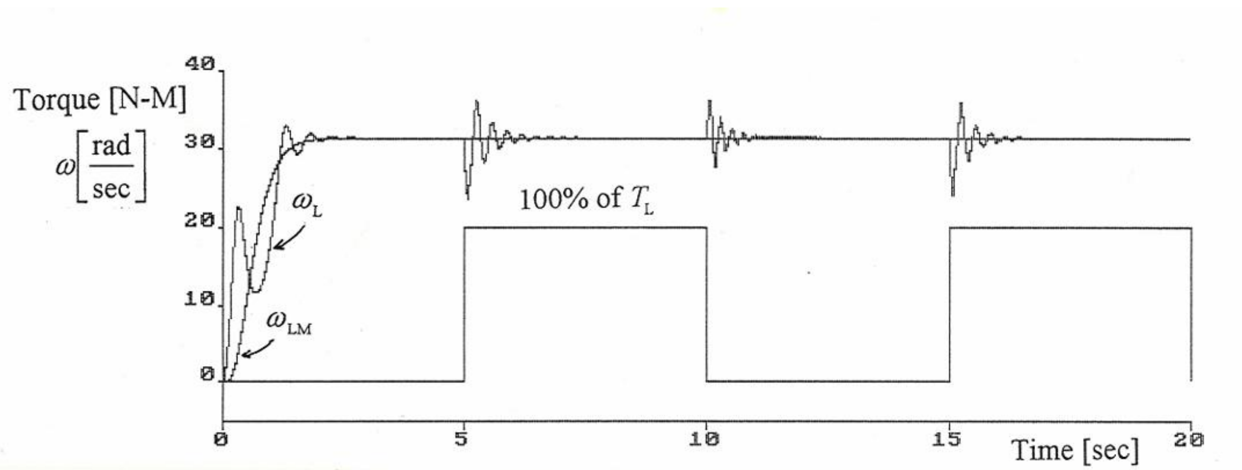

Figure 5. (a) Simulation response with a 100% load.

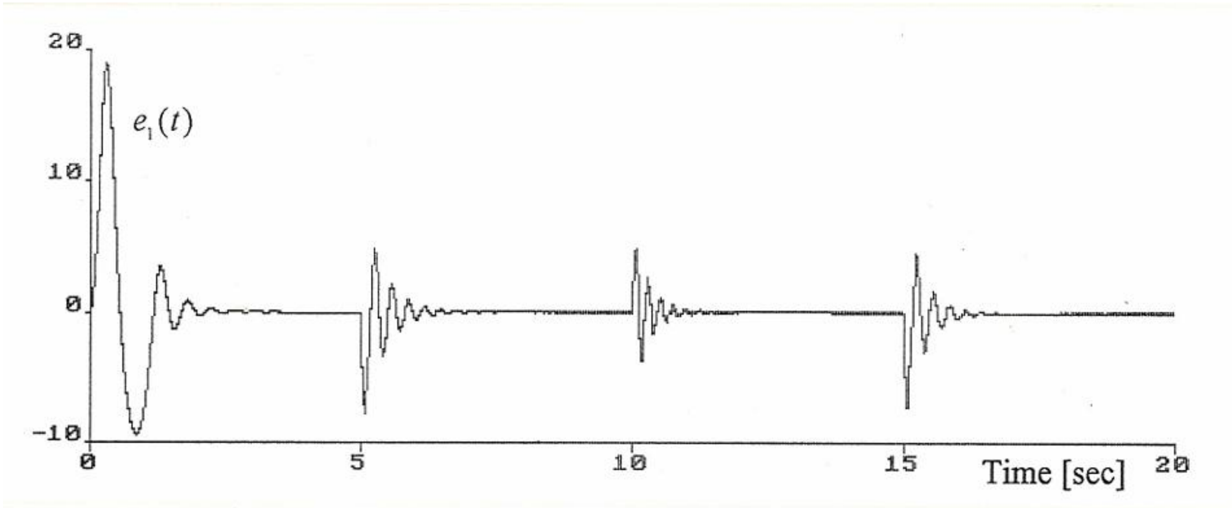

Figure 5.(b). The error $e_1(t)$ when the system is in the loading condition (100% load).

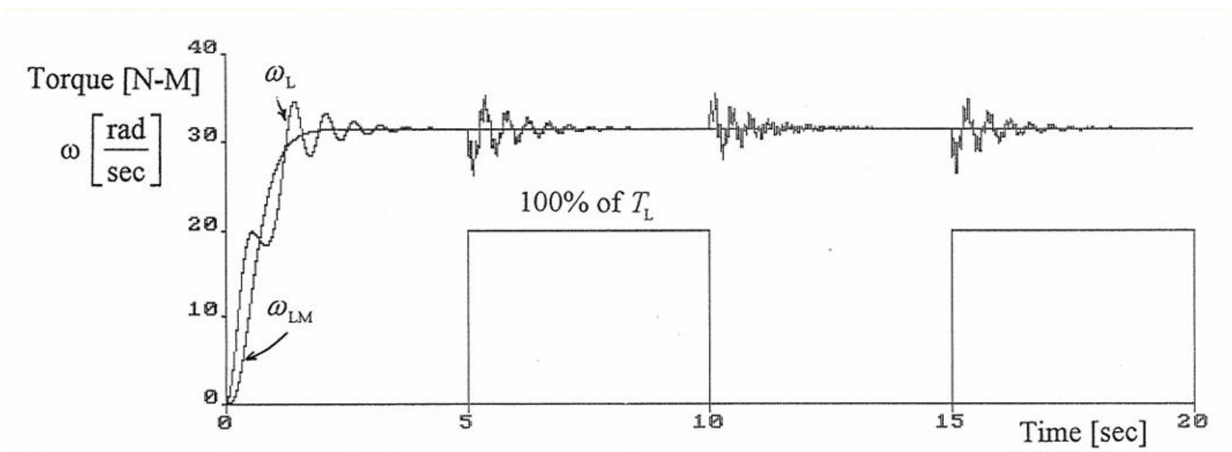

Figure 6. The output of the system when the parameter J_M is increased by a factor of 5.

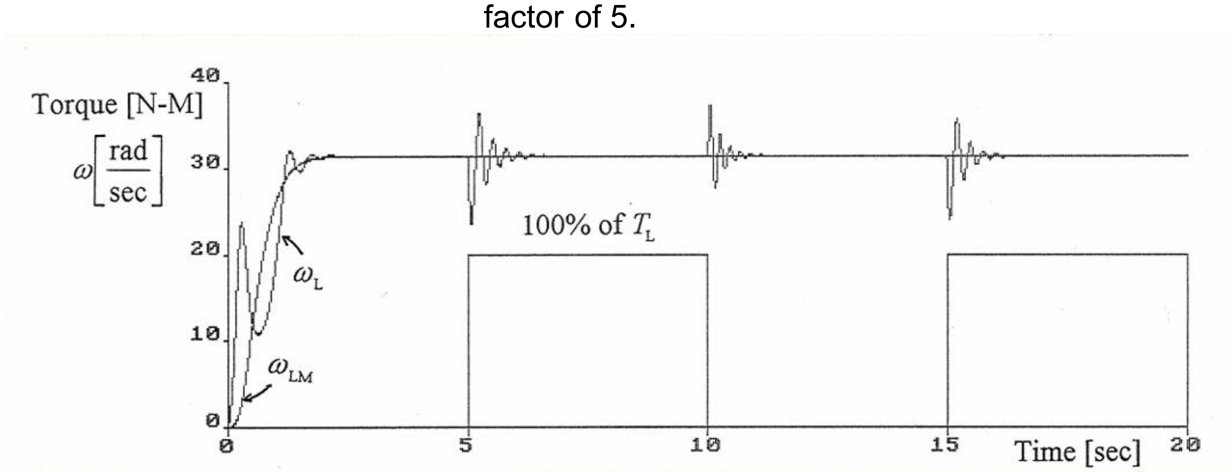

Figure 7. The output response of the system when the parameter J_M is decreased by a factor

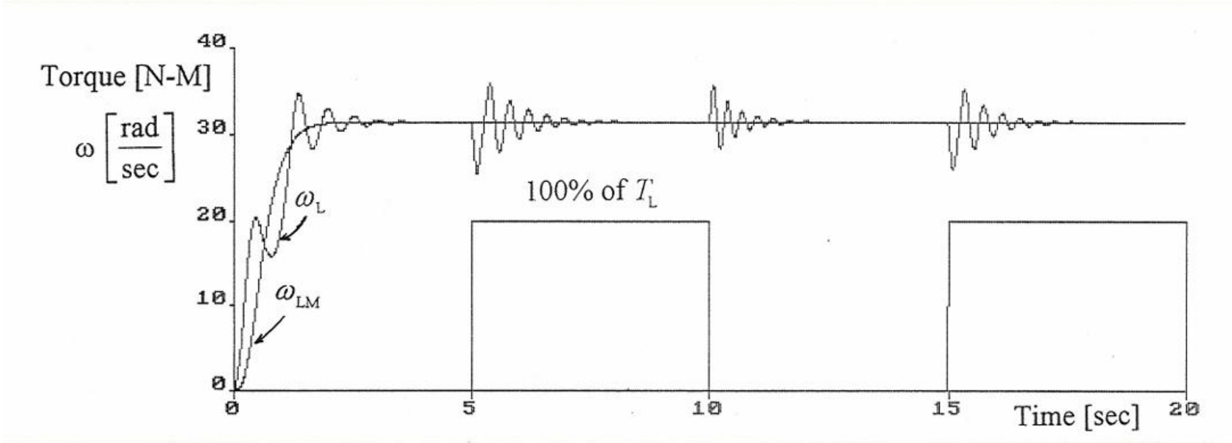

Figure 8. The output response of the system when the parameter J_L is increased by a factor 5.

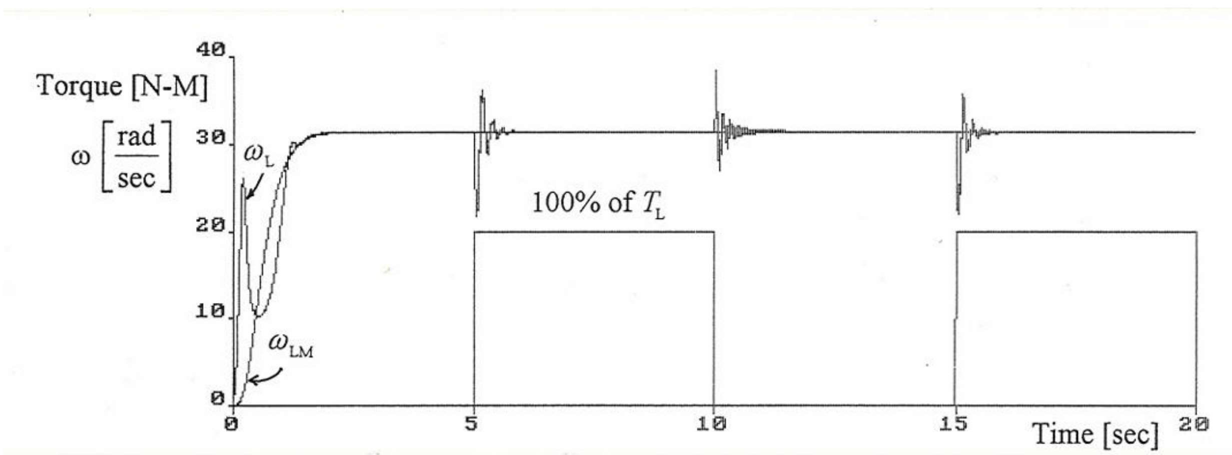

Figure 9. The output response of the system when the parameter J_L is decreased by a factor 5.

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